B.Sc (Math) part - II
paper - III
Topic :- Notion of group, Abelian group and non-abelian group

Def :— Let G be a non empty set and o be the binary operation on G. Then the set G together with the operation o denoted by (G, o) is called a group iff the following condition satisfied

G_1 : $\forall\ a, b \in G$ then $aob \in G$ (Closure)

G_2 : $\forall\ a, b, c \in G$ then $(aob)oc = ao(boc)$ (associative law)

G_3 : There exists an element e of G such that $aoe = eoa = a$ for all element $a \in G$ (Existence of identity) Then element e is called an identity element

G_4 : For each element $a \in G$ there exists an element a' of G such that $aoa' = a'oa = e$ (Existence of inverse)

The element a' (a^{-1}) is called inverse of a in G

~~Thus If Se~~

Abelian Group :—

Def :— A Group (G, o) is called abelian group If the Commutative law hold for the operation

(G_5) Commutative law :- The Commutative law for the operation 'o' holds i.e $aob = boa$ for all $a, b \in G$

Ex:-① prove that the set of integer is an abelian group under addition.

Soln:- Let I be the set of integers that is $I = \{ \dots -3, -2, -1, 0, 1, 2, 3 \dots \}$

G_1: the sum of two integers is an integer Hence if $a, b \in I$ then $a+b \in I$

G_2: Addition of integer is associative Hence if $a, b, c \in I$ then
$a+(b+c) = (a+b)+c$

G_3 the identity of I is 0 for $a+0 = 0+a = a$ for all $a \in I$

~~G_4 The identity inverse of I is~~

G_4: The inverse of $a \in I$ is $-a \in I$ for $a+(-a) = (-a)+a = 0$

Thus all the group postulate are satisfied and hence I is a group Moreover I is an Abelian group since addition in I is commutative.

Ex-2 prove that the set of integers is not a group under multiplication.

Soln:- ① Let I be the set of integer Hence if $a, b \in I$ then $a \cdot b \in I$

(ii) The multiplication of integ

is associative hence $a, b, c \in I$ then $a \cdot (b \cdot c) = (a \cdot b) \cdot c$

(iii) The identity of I is $1 \in I$ for $a \cdot 1 = 1 \cdot a = a$ for every $a \in I$.

(iv) The inverse of $a \in I$ is $\frac{1}{a}$ for $a \cdot \frac{1}{a} = 1$ But $\frac{1}{a} \notin I$.

Thus forth postulate is not satisfied and hence I is not a group under multiplication.

Ex:- prove that the four fourth root of unity i.e the set $\{1, -1, i, -i\}$ is abelian group w.r.to multiplication

Soln:- $(G, \times)$ is group for

(i) closure law hold:- it is clear from the following multiplication table

$\times$	1	-1	i	$-i$
1	1	-1	i	$-i$
-1	-1	1	$-i$	i
i	i	$-i$	-1	1
$-i$	$-i$	i	1	-1

(ii) Associative law hold

(iii) identity element 1 exists

(iv) inverse element exists as follow $(1)^{-1} = 1, (-1)^{-1} = -1, (i)^{-1} = -i, (-i)^{-1} = i$

Also G is an Abelian group since the table is symmetrical about the main digonal which begins from the left hand corner.

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